MATH 147 QUIZ 3 SOLUTIONS

1. For the function $f(x,y) = \begin{cases} \frac{2x^2y^2}{\sqrt{x^2+y^2}}, & \text{if } (x,y) = (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$, verify that partial derivative with respect to x is continuous at (0,0). Then use the limit definition to show that $f(x,y)$ is differentiable at (0,0).

   First, we calculate f_x around not at the origin. This is $f_x = \frac{\sqrt{x^2+y^2}4xy^2 - 2x^3y^2(x^2+y^2)^{-1/2}}{x^2+y^2}$. Then, to check the behavior of this function at $(0,0)$ we take the limit as it approaches the origin. We use a polar substitution to do so. We have

$$\lim_{(x,y)\to(0,0)} f_x(x,y) = \lim_{r\to 0, \theta\in\mathbb{R}} \frac{4r^4(\cos(\theta)\sin^2(\theta)) - 2r^5\cos^3(\theta)\sin^2(\theta)r^{-1}}{r^2} = \lim_{r\to 0,\theta\in\mathbb{R}r} r^2\left[4\cos\sin^2 - 2\cos^3\sin^2\right] = 0.$$

   On the other hand, we can take the limit definition of derivative to find the actual value of $f_x(0,0)$. We have that

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{2h^2 0}{h} - 0}{h} = \lim_{h\to 0} 0 = 0.$$

   As the limit of the function is equal to its value of the function at the origin, we say that f_x is continuous at $(0,0)$.

2. Find $DF(2,3,1)$ for the function $F(x,y,z) = (x^2y^3z, e^{xy^2z^3}, \cos(xyz))$.(5 points)

   By taking the correct derivatives of the component functions with respect to the variables, we get that

$$DF(x,y,z) = \begin{pmatrix} 3xy^3z & 3x^2y^2z & x^2y^3 \\ y^2z^3e^{xy^2x^3} & 2xyz^3e^{xy^2x^3} & 3xy^2z^2e^{xy^2x^3} \\ -yz\sin(xyz) & -xz\sin(xyz) & -xy\sin(xyz) \end{pmatrix}$$

   which leads us to

$$DF(2,3,1) = \begin{pmatrix} 108 & 108 & 108 \\ 9e^{18} & 12e^{18} & 54e^{18} \\ -3\sin(6) & -2\sin(6) & -6\sin(6) \end{pmatrix}.$$